

Jastrow-Luttinger Fractional Liquids

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In this paper, we present a description of Haldane's Luttinger liquid which parallels Laughlin's theory of the Fractional Quantum Hall (FQH) incompressible fluid, both exhibiting similar ground states as well as fractional excitations. These two non-Fermi liquids are instances of a generic structure for low-dimensional quantum liquids which we propose to dub Jastrow-Luttinger Fractional Liquids. An important feature of such liquids is the complete fractionalization of the parent particle. In particular, in both one and *two* dimensions spin-charge separation can be achieved and is indeed suggested to occur for unpolarized quantum Hall systems both at the edge and in the bulk.

The goal of this paper is to present strong arguments supporting the existence of a generic theoretical structure for a class of low dimensional condensed matter systems, which we propose to dub Jastrow-Luttinger fractional liquids. Among these systems we find two prominent non-Fermi liquids namely the Luttinger liquid (LL) which describes a one dimensional metal [1], and in two dimensions the incompressible fluid of the Fractional Quantum Hall Effect (FQHE) [2]. The fact that these two quantum liquids are related is well known: indeed edge excitations of a quantum Hall sample are believed to be described by a chiral Luttinger liquid [3], a variant of the usual Luttinger liquid. Yet, it will perhaps come as a surprise to learn that the connection between the two fluids is more fundamental. Indeed the wavefunctions of both the ground state and excitations of the gaussian boson hamiltonian describing the LL are the precise 1D analogs of the variational states considered in the FQHE: the ground state is a Jastrow-Laughlin (JL) wavefunction, charged excitations have the functional form of Laughlin's quasiparticles, while neutral excitations are density fluctuations in complete analogy to the single-mode approximation (SMA) approach to the FQHE [4,5]. Our claim is consistent with the result obtained by Fradkin et al [6] who used a general field theoretical formalism to show that the square modulus of the Thirring model ground state functional has the Jastrow-Laughlin form. We will actually see that the full wavefunctions of both the ground state and its excitations can be derived in a very elementary manner.

We use the term "fractional" to describe Jastrow-Luttinger liquids because they exhibit a *fractionalization* of the basic constituent particles: the quantum numbers of these particles (electrons, bosons or spins) have completely vanished from the spectrum of elementary excitations [7]. This is first evidenced at the level of charged excitations which no longer carry a unit charge with respect to the ground state (e.g. the fractional charges of the FQHE, the spinon of the 1D Heisenberg model). Furthermore when we add internal quantum numbers to

the $U(1)$ charge, the hamiltonian and neutral excitations show a separation into independent collective modes each separately carrying part of the quantum numbers of the basic particle; this separation for the hamiltonian carries over to the ground state and charged excitations wavefunctions, and to all quantum averages: they can be factorized into dynamically independent parts. We usually have independent charge and spin modes -that is spin-charge separation as in the LL- if some simple symmetry requirement (derived below) is obeyed; if the condition is not fulfilled, spin-charge separation is not realized but a more general *quantum numbers separation* still exists. An instance will be given with the one dimensional Hubbard model in a magnetic field. After a discussion of the 1D case with the LL, we will turn to the 2D case. We will show that spin-charge separation may occur in quantum Hall samples whenever a spin unpolarized ground state is achieved, for states describable by a Halperin wavefunction (a multicomponent generalization of the usual Laughlin state [8]): this is experimentally relevant to fillings such as $\nu = 8/5$. We will also define the idea of *pseudo-confinement* which is required for a correct understanding of fractionalization. After a discussion of edge states in the FQHE whose microscopic relation to Laughlin's bulk theory we try to clarify, we briefly discuss issues in quantum magnetism such as the spin liquid, arguing for its existence.

Heisenberg Model.

Our initial clue came from the observation that both the one dimensional Heisenberg chain and the XY model - which are Luttinger Liquids - have ground states (approximate in the first case) with the Jastrow-Laughlin (JL) form: $\psi_\lambda(\{r_1, \dots, r_N\}) = \prod_{i < j} |z_{ij}|^\lambda$, where the $\{r_i\}$ are the positions of N down spins (in the hard-core boson representation) on a chain of length $L = 2N$ with periodic boundary conditions, and $z_i = e^{i2\pi r_i/L}$ ($\lambda = 1$ for the XY model, while $\lambda = 2$ for the Heisenberg chain). The approximate ground state for the Heisenberg chain yields a very good variational energy [9] and its correla-

tors have the correct large distance properties: it is actually the exact ground state of a $1/r^2$ exchange spin model, the Haldane-Shastry chain which belongs to the universality class of the Heisenberg model [10]. The Calogero-Sutherland model [11], a $1/r^2$ interaction model related to the Haldane-Shastry chain was shown through finite-size scaling methods combined to conformal theory to be a Luttinger liquid [12]: again its exact ground state has the Jastrow-Laughlin form. These remarks led us to investigate whether Jastrow-Laughlin wavefunctions might be suitable variational states for arbitrary Luttinger liquids. We tested this idea on the anisotropic Heisenberg chain:

$$\begin{aligned} H_{xxz}(\Delta) &= J \sum_i \frac{1}{2} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + \Delta S_i^z S_{i+1}^z \\ &= J \sum_i \frac{-1}{2} (b_i^+ b_{i+1} + b_i b_{i+1}^+) + \Delta n_i n_{i+1} \quad (1) \end{aligned}$$

where in the second line we use a hard-core boson representation for $S = 1/2$ spins [13]. (A rotation around the z axis has also been performed for spins on odd sites.) J is the exchange integral and the anisotropy Δ is chosen to vary in the range $[-1; 1]$ where the above model is known to be a LL [14]. We then consider :

$$\psi_\lambda(\{r_1, \dots, r_N\}) = \prod_{i < j} |z_{ij}|^\lambda \quad (2)$$

and find indeed that Ψ_λ yields excellent variational energies per site correct to the third decimal place when compared to the exact ground state energies obtained from the Bethe ansatz solution. It is known that the Luttinger liquid parameter K which controls the anomalous exponents is given for a given anisotropy Δ by $\Delta = -\cos(\frac{\pi}{2K})$ [14]. We find that the best values of λ for a given Δ satisfy $\lambda = 1/K$. This is expected since a 1D plasma analogy similar to the one introduced in [2] gives that the transverse spin-spin correlator for Ψ_λ decays as $1/r^{\lambda/2}$ at large distance while the LL theory predicts an exponent $1/2K$ [15]. (In one dimension the fictitious plasma is the well-known Dyson gas of random matrices [16].)

Ground state and neutral excitations of the LL.

The previous result suggests that all Luttinger liquids might indeed be liable to a variational Jastrow-Laughlin description, which leads us to address the question of the precise relation between such a variational approach and the LL formalism. In other words we would like to compare our Jastrow wavefunction with the ground state of the boson LL hamiltonian. The task is easy; the gaussian boson hamiltonian is just a sum of harmonic oscillators so that the determination of the ground state and of its excitations is trivial:

$$H_B = \frac{u}{2} \int_0^L dx K^{-1} (\nabla \Phi)^2 + K (\nabla \Theta)^2 \quad (3)$$

where Θ and $\Pi = \nabla \Phi$ are canonical conjugate boson fields; u and K are parameters giving respectively the velocity of the harmonic wave and controlling anomalous exponents. We recall that the LL hypothesis states that in one dimension any gapless hamiltonian H (for bosons, as well as for fermions and spins) will admit an effective low-energy description in terms of the gaussian hamiltonian H_B for a suitable choice of the parameters u and K (these can be determined by finite-size scaling methods or by comparison to exact solutions whenever available). The mapping is completed through the relations $j = \frac{1}{\sqrt{\pi}} \nabla \Theta$ and $\delta \rho = \frac{1}{\sqrt{\pi}} \nabla \Phi$ where j and $\delta \rho$ are respectively the particle current and a density fluctuation about a mean value ρ_0 , plus the definition of the particle creation operators: $\Psi_B^+ = \rho^{1/2} \exp(i\sqrt{\pi}\Theta)$ for bosons, and $\Psi^+ = \Psi_B^+ (\exp(ik_F r + i\sqrt{\pi}\Phi) + \exp(-ik_F r - i\sqrt{\pi}\Phi))$ for fermions. The Fourier-transform of H_B is:

$$\begin{aligned} H_B &= \frac{u}{2} \sum_{q \neq 0} K^{-1} \Pi_q \Pi_{-q} + K q^2 \Theta_q \Theta_{-q} \\ &\quad + \frac{\pi u}{2L} \left(\frac{\hat{Q}^2}{K} + K \hat{J}^2 \right) \quad (4) \end{aligned}$$

where q is quantized as $q_n = 2\pi n/L$, $\hat{Q} = \hat{N} - N_0$ counts particles from N_0 the ground state particle number and $\hat{J} = \int j$ is a particle current. The ground state wavefunction of an harmonic oscillator is a gaussian and is obtained for $N = N_0$ and $J = 0$: $\Psi_0 = \exp(-\frac{1}{2K} \sum_{n \neq 0} \frac{1}{|q_n|} \Pi_n \Pi_{-n})$; returning to the original variables through $\Pi_q = \sqrt{\pi/L} \rho_q = \sqrt{\pi/L} \sum_i \exp(iqr_i)$, we easily find that Ψ_0 is nothing but a Jastrow-Laughlin wavefunction!

$$\psi_0(\{r_1, \dots, r_{N_0}\}) = \prod_{i < j} |z_{ij}|^{1/K} \quad (5)$$

This is the correct form if we consider bosons; for fermions, we note that the fermion creation operator is deduced from that of the boson by a Jordan-Wigner phase factor multiplication which ensures antisymmetrization (see Appendix); this means that reducing H to H_B for fermions is achieved by first making a singular gauge transformation which converts the particle statistics to a bosonic one. In the end, this transformation must be undone and we obtain $\psi_0^F(\{r_1, \dots, r_N\}) = \prod_{i < j} (z_{ij}) |z_{ij}|^{1/K-1} \exp(ik_F \sum r_i + c.c.)$. This derivation of the ground state follows exactly the same lines as that for the bosonic Landau-Ginzburg theory for the FQHE [17]. Neutral excitations above the ground state (for which $Q = 0$ and $J = 0$) are Hermite polynomials:

$$|n_{q_1}, n_{q_2}, \dots, n_{q_p}\rangle = \prod_{s=1}^p H_{n_{q_s}} \left(\sum_i z_i^{n_{q_s}} / \sqrt{LK|q_s|} \right) |\Psi_0\rangle \quad (6)$$

As usual for a harmonic oscillator they are obtained from ladder operators which are here: $a_q = \frac{\Pi_q}{\sqrt{2K|q|}} - i\sqrt{\frac{K|q|}{2}}\Theta_q$. For instance, $\Psi_{n_q} \propto \rho_q \Psi_0$. We note that this is just the excitation predicted by single-mode approximation (SMA) theories, which were first considered by Feynman in the context of superfluid ^4He [18]. These excitations are the collective modes of the LL.

In this part we have derived the wavefunctions of the LL boson hamiltonian for the ground state and its neutral excitations, which we have found to be isomorphic to those considered for the FQHE. We will now show that this parallel holds also for charged excitations.

Charged excitations in the LL.

The LL does not support only density wave excitations : it is well known indeed from Bethe Ansatz that there are spinons in the 1D Heisenberg model (spin 1/2 excitations), and additionally holons in the Hubbard model. The form of the LL ground state invites us to write down the following variational quasihole wavefunctions, in complete analogy with the FQHE:

$$\Psi_{z_0}(x_i) = \prod_{i=1}^N (z_i - z_0) \prod_{i < j} |z_{ij}|^{1/K} \quad (7)$$

As in two dimensions, from the plasma analogy we can show that Ψ_{z_0} carries a charge K :

$$|\Psi_{z_0}|^2 = \exp 1/K \int \int dy dy' [\rho(y) + K\delta(x-y)] \ln \left| \sin \frac{\pi}{L}(y-y') \right| [\rho(y') + K\delta(x-y')] \quad (8)$$

For $K = 1/2$ (relevant to the Heisenberg model), that object has exactly the spin expected for the spinon. We will now show that this guess is actually the correct answer: a LL with parameter K has excitations carrying charges which are integer multiples of K .

Excitations can be classified according to the zero modes \hat{Q} and \hat{J} defined in eq.(4) which define (topological) charge sectors $\{Q, J\}$: in each of these sectors states are obtained from a lowest energy state by repeated application of the ladder operators a_q^\pm which do not change the particle numbers. The identification of charged excitations (for which $N \neq N_0$ and $J \neq 0$) is a standard operation in conformal field theory (CFT), where they are obtained from so called primary operators which generate the lowest energy eigenstates in a given charge sector. For the gaussian theory, (a $c = 1$ CFT), they are well known to be vertex operators [19,20]:

$$V_{\alpha,\beta}(x) =: \exp i \left(\sqrt{\pi}\beta\Phi(x) + \sqrt{\pi}\alpha\Theta(x) \right) : \quad (9)$$

($::$ denotes normal ordering) which obey the commutation relations $[\hat{Q}, V_{\alpha,\beta}] = \alpha V_{\alpha,\beta}$ and $[\hat{J}, V_{\alpha,\beta}] = \beta V_{\alpha,\beta}$. We interpret these excitations as longitudinal fluctuations of

the superfluid phase Θ and the $\exp(i\Phi)$ term as describing vortices. $J = \beta$ and $Q = \alpha$ are therefore topologically quantized and are respectively the vortex circulation and the number of quasiparticles. They are integers which we note now as $J = n$ and $Q = m$. Note that in a dual language which exchanges vortex and particle variables, quasiparticle excitations become (dual) vortices and the quantization of Q is simply the quantization of the circulation for the dual vortex.

In each topological sector the charged excitation with lowest energy is $V_{m,n}(z=0)$ with energy $\pi u(Q^2/K + KJ^2)/2L$. For other values of the parameter z , $V_{m,n}$ describes a coherent state living in the same charge sector. As is often pointed out to demonstrate the non-Fermi liquid nature of the LL, the electronic Green functions (i.e. for the $V_{m,n}$'s which in the non-interacting problem represent $Q = m$ (bare) particles carrying a current $J = n$) do not develop quasiparticle poles: it is believed that only neutral excitations are fundamental due to a decay of (bare) electrons. The fate of the electron is more subtle however.

The physical meaning of such operators is revealed when we go to first quantization. Let us consider for instance $V_{1,0}$; using the identity (for a review of the bosonization technique, see [21]): $\Theta(x) = \sum_q \theta_q e^{iqx}/\sqrt{L} = \sum_q \text{sgn}(q)\phi_q e^{iqx}/\sqrt{L} = -i/\pi \int_0^L dy \ln \left| \sin \frac{\pi}{L}(x-y) \right| \nabla \Phi(y)$ we see that in first quantization:

$$V_{1,0} = \exp \int_0^L \ln \left| \sin \frac{\pi}{L}(x-y) \right| \rho(y) = \prod_{i=1}^{N_0} |z_i - z| \quad (10)$$

Similarly, $V_{0,1} = \prod_{i=1}^{N_0} (z_i - z)/|z_i - z|$ and $V_{1,1}(x)\Psi_0(x_i) = \prod_{i=1}^N (z_i - z) \prod_{i < j} |z_{ij}|^{1/K}$. Therefore $V_{1,1}\Psi_0$ is precisely the quasihole wavefunction Ψ_{z_0} introduced above! This again is completely analogous to the FQHE. We remark that $V_{0,1}$ precisely describes a phase singularity which justifies its identification as a vortex. The elementary charge quantum was determined in eq.(8) through plasma analogy and charges are therefore integer multiples of K : $Q_c = QK$ where $Q = m$ is an integer (this is the number of quasiparticles).

Quasi-electrons then are described by vertex operators with negative m : an example is $V_{-1,-1}$, the effect of which amounts to dividing by a factor $\prod_{i=1}^N (z_i - z)$; we note that the description of anti-vortices in the LL theory is restricted to a region excluding the core due to a divergence when a particle comes close to z . This is a difficulty shared by the effective gaussian Chern-Simons Landau-Ginzburg theory in 2D [17] which does not affect however the correctness of the description at long-distance.

The LL has a chiral symmetry. This has the additional consequence that the hamiltonian can be separated into chiral components $H_B = H_+ + H_-$:

$$H_\epsilon = \frac{u}{2} \sum_{\epsilon q > 0} K^{-1} \Pi_q \Pi_{-q} + K q^2 \Theta_q \Theta_{-q} + \frac{\pi u}{LK} \hat{Q}_\epsilon^2 \quad (11)$$

where $\hat{Q}_\epsilon = (\hat{Q} + \epsilon K \hat{J})/2$ are chiral charges which count the number of particles within each chiral sector: $\hat{Q} = \hat{Q}_+ + \hat{Q}_-$; eigenvalues have the form: $Q_\epsilon = (m + \epsilon K n)/2$ where n and m are arbitrary integers. Although there is a chiral separation of the hamiltonian, it must be noted that the chiral charges of the allowed excitations can not vary independently: they are constrained by the relation $Q_\epsilon = (m + \epsilon K n)/2$. Since charged excitations mix chiralities, one might be tempted to infer that chiral excitations are confined (not separated): yet this is not correct. Rather one has a *pseudo-confinement*: although the true charged elementary excitations (the $V_{m,n}$) do appear as chiral composites, their chiral components are still free, dynamically independent. The constraint $Q_\epsilon = (m + \epsilon K n)/2$ acts like a selection rule which has no bearing on the chiral separation. We want to stress this point which will come out again in the discussion of spin-charge separation. In the context of spin-charge separation, that pseudo-confinement will appear with charged excitations carrying both charge and spin although spin-charge separation is indeed realized. The origin of these selection rules is the topological quantization of Q and J the number of quasiparticles and their vorticity.

The fact that excitations come with both chiralities according to the above constraint is very similar to the familiar topological constraint on spinons in the Heisenberg chain which must come in pairs since they have a spin $1/2$ [23]; they nevertheless are free particles. Since physical probes can only involve integer numbers of particles, the only observable variations of the spin and of the number of particles ΔS_z and ΔN must be integer-valued. This means that in a LL only charged excitations carrying rational charges can be observed. Irrational charges K are allowed only as parts of globally neutral complexes.

It is often stated that for $K < 1$ (resp. > 1), the LL describes repulsive (resp. attractive) interactions. A simple examination of (5) allows a simple derivation of that result: when interactions are repulsive, the ground state will develop higher-order zeros to keep particles further apart. Now note that in the repulsive case the charge of the quasiparticle will indeed be a fraction of the electron's unit charge; but in the attractive case, our quasiparticle has a charge larger than that of the electron: this reflects the attractive nature of the interactions though the result is probably surprising.

We make additional remarks: (i) The boson hamiltonian displays a superfluid rigidity for hard-core bosons. There is indeed a very striking parallel with theories of ^4He : we have a Jastrow ground state, vortex excitations (the quasiparticles) and phonons (the neutral collective modes) [18]. These hard-core bosons are obtained after a singular gauge transformation on fermions: we

may as in the FQHE understand the superfluidity as a hidden off-diagonal long-range order [28]. (ii) The fractional excitations $V_{m,n}$ obey conventional exchange statistics which can be shown to be: $\pi n m$ [22], but anyons may however appear if we consider generalizations of the LL to fields with conformal spin $S = (2n + 1)/2$; (this will be discussed when we consider edge states). (iii) Still, quasiparticles obey fractional exclusion statistics [24] with statistics parameter K ; exclusion statistics is characterized by the fact that one state can be occupied by at most one fermion or by any number of bosons, but by $1/g$ particles obeying exclusion statistics with statistics g . For instance, for $V_{1,1}$ quasiparticles with charge $K = 1/q$, statistics is $1/q$ since $\Psi_0(x_1 \dots x_{N+1}) = [V_{1,1}(x_{N+1})]^q \Psi_0(x_1 \dots x_N)$. (iv) A difficult problem in LL is the determination of the parameters (u, K) ; their numerical evaluation through the variational principle makes it a simple matter: integrals with Jastrow-Laughlin functions can be done straightforwardly with simple Metropolis algorithms. (v) Neutral collective excitations which are bare particle-hole pairs may also be viewed as Laughlin quasiparticle-quasihole pairs.

The structure of the excitation spectrum is summarized as follows: it is given by the set of integers $\{N_0, Q, J\}$ where for a given number of particles N_0 we classify excitations in topological sectors $\{Q, J\}$. In each sector there are topologically neutral excitations which are density modulations. States in the sector $\{N_0, Q, J\}$ carry a charge $N = N_0 + KQ$; however $Q \neq 0$ is not allowed if K is irrational since in the Fock space $F = H(N = 0) \oplus H(N = 1) \oplus \dots$ only states with a global integer charge exist. Transitions to the sector $\{N_0 + n, Q, J\}$ cannot therefore be understood in terms of quasiparticle excitations from the sector $\{N_0, Q, J\}$ unless the integer n is a multiple of K .

In summary we have shown in this part the novel result that Luttinger liquids -among which we find important models such as the Heisenberg chain or the Hubbard model- sustain very peculiar charged excitations which carry indeed *anomalous* charges (i.e. non-integer in general), obey exclusion statistics and which are just the 1D counterparts of the well-known Laughlin quasiparticles. (A difference however is that charges in the FQHE are rational numbers.) For instance the anisotropic Heisenberg model -eq.(1)- has spin excitations carrying a spin $K = \frac{\pi}{2} / \arccos(-\Delta)$. As we vary the anisotropy Δ from 1 to 0, we observe that the anomalous spin will vary from $1/2$ (the isotropic chain) to 1 (*XY* model) in a *continuous* manner. For ferromagnetic anisotropies ($\Delta < 0$) the spin is larger than one (the interaction is indeed attractive).

We conclude this part by a theorem: "the wavefunctions of the exact eigenstates of the LL boson hamiltonian are a Jastrow-Laughlin wavefunction for the ground state, Laughlin quasiparticles for the charged excitations and Bijl-Feynman phonons for the neutral excitations."

Quantum numbers separation.

We now add internal quantum numbers to the LL, and for definiteness focus on usual spins, considering the following two-component wavefunctions:

$$\psi_0(\{r_i, \sigma_i\}) = \prod_{i < j} |z_{ij}|^{g_{\sigma_i, \sigma_j}} \quad (12)$$

where \hat{g} the charge matrix is a 2×2 symmetric matrix (\hat{g} describes the charges of the classical plasma associated with Ψ_0). For fermions we antisymmetrize the wavefunction as follows: $\psi_F(\{r_i, \sigma_i\}) = \prod_{i < j} |z_{ij}|^{g_{\sigma_i, \sigma_j} - \delta_{\sigma_i, \sigma_j}} (z_{ij})^{\delta_{\sigma_i, \sigma_j}} e^{i\frac{\pi}{2} \text{sgn}(\sigma_i - \sigma_j)}$ (the exponential which ensures antisymmetrization between different species is known as a Klein factor). We want to recover spin-charge separation which is believed to be a characteristic of the ("spinful") LL. Quite often a problem difficult to handle in terms of certain variables may become simple if one changes to dual variables: the highly collective nature of the LL (anomalous charges can only exist on that account) suggests to switch from individual particle coordinates to collective ones -namely densities. Spin-charge separation is then indeed readily apparent. We state that ψ_0 is the exact ground state of the following hamiltonian for arbitrary velocities u_i :

$$H(\hat{g}) = \sum_{i=1}^2 \frac{u_i}{2} \int_0^L dx K_i^{-1} (\nabla \Phi_i)^2 + K_i (\nabla \Theta_i)^2 \quad (13)$$

where the fields Φ_i are related to the densities ρ_\uparrow and ρ_\downarrow by $\rho_i = \nabla \Phi_i / \sqrt{\pi}$ and $\rho_\sigma = P_{\sigma i} \rho_i$ where P is the unitary matrix which puts \hat{g} to diagonal form with eigenvalues K_1^{-1} and K_2^{-1} , i.e. $P^{-1} \hat{g} P = \text{diag}(1/K_1, 1/K_2)$. We rewrite ψ_0 as:

$$\begin{aligned} & \exp 1/2 \int dy dy' \rho_\sigma(y) g_{\sigma\tau} \ln \left| \sin \frac{\pi}{L} (y - y') \right| \rho_\tau(y') \\ & = \exp \sum_i 1/2 K_i \int dy dy' \rho_i(y) \ln \left| \sin \frac{\pi}{L} (y - y') \right| \rho_i(y') \end{aligned} \quad (14)$$

which proves the above statement and makes a separation into normal modes ρ_1 and ρ_2 manifest. We now see that spin-charge separation can only occur if there is a Z_2 symmetry between up and down spins for \hat{g} , i.e. if \hat{g} has the form $\begin{pmatrix} \lambda & \mu \\ \mu & \lambda \end{pmatrix}$. Then $H(\hat{g})$ is the usual spin-charge separated boson hamiltonian with LL parameters $K_\rho = \frac{1}{\lambda + \mu}$ and $K_\sigma = \frac{1}{\lambda - \mu}$. Quite generally, if the gaussian hamiltonian breaks this invariance, so will the ground state since it is built from the normal modes of the hamiltonian; in that case, instead of getting spin-charge separation one has a more general "quantum numbers separation"; normal modes will each carry parts of the quantum numbers of the electron in a proportion *fixed* in time, and will factorize into dynamically independent parts for all observables, which is a consequence of the separability of the hamiltonian (at $T = 0$ this can be seen equivalently as

separability of the ground state). That separation which is highly non-trivial in terms of particles stems therefore from the rather trivial statement that with N internal degrees of freedom, there will be N normal modes. (Separation into chiral components -see (11)- is exactly similar.)

As an illustration of quantum numbers separation, we discuss the Hubbard model in one dimension. For strong repulsion U , the Bethe Ansatz ground state is known to factorize in spin and charge parts [25]: $\Psi \propto \prod_{i < j} (z_{ij}) \Psi_H$ where Ψ_H the spin part is related to the Heisenberg chain Bethe Ansatz wavefunction, and the first part (the charge part) is just a Slater determinant. Are we able to extract the LL parameter K_ρ ? Relating this form to the Jastrow-Laughlin one, we can indeed read off $K_\rho = 1/2$! In a magnetic field, from the exact solution, it was argued [26] that spin-charge separation is not realized, and that the Hubbard model is a semi-direct product of two $c = 1$ CFT's. A dressed charge matrix Z describing renormalized charges of excitations was introduced. That formalism can be related to the LL: it can be shown that with the choice $\hat{g} = M^T M$ where M is related to Z through $M = \begin{pmatrix} z_{cc} - z_{sc} & z_{sc} \\ z_{cs} - z_{ss} & z_{ss} \end{pmatrix}$ where the z_{ij} are Z matrix elements, the anomalous exponents predicted by Bethe Ansatz (plus CFT) are completely reproduced by the two-component LL corresponding to \hat{g} [27]. This analysis yields a \hat{g} matrix which breaks the Z_2 symmetry between up and down spins, confirming the fact that spin-charge separation no longer occurs when a magnetization sets in: but the model still maps onto a LL, i.e. to $H(\hat{g} = M^T M)$ which is the direct product of two $c = 1$ CFT. We conclude that in a magnetic field the one dimensional Hubbard model exhibits a quantum number separation (though no longer spin-charge separation).

Charged excitations are again vertex operators:

$$V_{m,n,m',n'} =: e^{i\sqrt{\pi}(n\Phi_\uparrow + m\Theta_\uparrow + n'\Phi_\downarrow + m'\Theta_\downarrow)} : \quad (15)$$

where $J_\sigma = (n, n')$ and $Q_\sigma = (m, m')$ are integer bare charges. Their exchange statistics is $\pi(mn + m'n')$ and is therefore conventional. We rewrite eq.(12) as:

$$\psi_0 = \prod_{\uparrow} |w_{ij}|^\lambda \prod_{\downarrow} |y_{ij}|^{\lambda'} \prod |w_i - y_j|^\mu \quad (16)$$

where $\{w_i, y_j\}$ are the positions of spins up and down respectively. (We have also set $g_{\uparrow\uparrow} = \lambda$, $g_{\downarrow\downarrow} = \lambda'$ and $g_{\uparrow\downarrow} = \mu$.) For example $V_{1,1,0,0} \Psi_0 = \prod_i (w_i - z_0) \Psi_0$. We can determine the charges e_\uparrow and e_\downarrow of the vertex operators by plasma analogy again: $|V_{Q_\sigma, J_\sigma} \Psi_0|^2 = e^{-U}$ where:

$$\begin{aligned} U &= \int \int [\rho_\sigma(y) + g_{\sigma\tau}^{-1} Q_\tau \delta(x_0 - y)] g_{\sigma\sigma'} \\ & \ln \left| \sin \frac{\pi}{L} (y - y') \right| [\rho_{\sigma'}(y') + g_{\sigma'\tau}^{-1} Q_\tau \delta(x_0 - y')] \end{aligned} \quad (17)$$

The charges are therefore: $e_\sigma = g_{\sigma\tau}^{-1}Q_\tau$. For instance for $V_{1,1,0,0}$, $Q_\sigma = (1,0)$ and the charges are: $(e_\uparrow, e_\downarrow) = \left(\frac{\lambda'}{\lambda\lambda' - \mu^2}, \frac{-\mu}{\lambda\lambda' - \mu^2}\right)$. These charges can also be determined algebraically from the zero modes of the normal densities ρ_i : in the normal basis, they are again $e_i = K_i Q_i$ (in the spinless case we identified the anomalous charge as KQ); going back to the spin basis, $e_\sigma = P_{\sigma i} K_i Q_i$. Since $Q_i = P_{\tau i} Q_\tau$ (because $\rho_i = P_{\sigma i} \rho_\sigma$), we obtain again $e_\sigma = P_{\sigma i} K_i P_{\tau i} Q_\tau = g_{\sigma\tau}^{-1} Q_\tau$. The vertex operators V_{Q_σ, J_σ} therefore carry a charge $q = e_\uparrow + e_\downarrow = \sum_{\sigma\tau} g_{\sigma\tau}^{-1} Q_\tau$ and a spin $S_z = \frac{1}{2}(e_\uparrow - e_\downarrow) = \sum_{\sigma\tau} \sigma g_{\sigma\tau}^{-1} Q_\tau / 2$. As discussed in the context of chiral separation, although charged excitations carry both anomalous charges $K_1 Q_1$ and $K_2 Q_2$, there is still quantum numbers separation: this again is not a true confinement, but rather a pseudo-confinement.

We specialize the discussion to the case of spin-charge separation: $\lambda = \lambda'$. We rewrite $V_{1,1,0,0} = \exp \int \rho_\uparrow(x) \ln(z - z_0) dx$ as:

$$e^{\frac{1}{2} \int \rho_c(x) \ln(z - z_0) dx} e^{\frac{1}{2} \int \rho_s(x) \ln(z - z_0) dx} \quad (18)$$

The two factors are dynamically independent due to spin-charge separation of the hamiltonian and correspond respectively to the holon and to the spinon. We can also express the holon as $h(z_0) = V_{1/2,1/2,1/2,1/2} = \prod_i (w_i - z_0)^{1/2} \prod_i (y_i - z_0)^{1/2}$ and the spinon as $s(z_0) = V_{1/2,1/2,-1/2,-1/2} = \prod_i (w_i - z_0)^{1/2} / \prod_i (y_i - z_0)^{1/2}$. They carry respectively charge and spin ($q = K_\rho, s = 0$) and ($q = 0, s = K_\sigma/2$). (In the $SU(2)$ symmetric case, $K_\sigma = 1$ and the spinon has spin $1/2$ as expected.) More generally $V_{m,n,m',n'}$ carries charge $q = (m + m')K_\rho$ and spin $s = (m - m')K_\sigma/2$. Note that the holon and the spinon are semions with statistics $\pi/2$. Due to pseudo-confinement however, the total number of holons and spinons is always even (as can be inferred from the expression for the charge and the spin of $V_{m,n,m',n'}$) and therefore neither parity P nor time reversal T are ever broken: we come to the surprising conclusion that we can have P and T breaking excitations although a (global) P and T breaking will never be observed. We stress that these pairs of holons and/or spinons are *not* bound. Such is not the case because they are dynamically independent: they depend on charge and spin densities which have independent dynamics due to the separation of the hamiltonian. The excitations appear as composites only because the total topological charges Q and J must be integers. The wavefunctions of the allowed charged excitations (and of the ground state) will factorize into independent charge and spin parts corresponding to the holons and spinons.

In summary we have found that spin-charge separation is a very natural property of Jastrow-Laughlin wavefunctions; we have generalized the usual spin-charge separation to a "quantum numbers separation" into inde-

pendent normal modes (which mix charge and spin in a proportion *fixed* in time). As in the spinless case we have argued again that there is a pseudo-confinement for charged excitations.

Jastrow-Luttinger Fractional Liquids.

We now systematically compare the LL and the FQHE, which leads us to formulate the concept of a Jastrow-Luttinger fractional liquid. (i) First of all, the ground states Ψ_0 have the same functional form and describe featureless liquids with a uniform density. (ii) Branches of neutral collective excitations correspond to $\rho_k \Psi_0$ [4]: they are density fluctuations above the liquid surface, and can be viewed therefore as bare particle-hole pairs or as anomalous charge quasiparticle-quasihole pairs. (iii) Charged excitations are solitons corresponding to bumps or holes at the surface of the liquid, i.e. Laughlin quasiparticles. (iv) In both cases, *gaussian* effective Landau-Ginzburg theories can be written: in the FQHE at filling $\nu = 1/(2n + 1)$ starting from a microscopic hamiltonian, after a Chern-Simons transformation one can derive an effective Landau-Ginzburg theory; after integration of the Chern-Simons gauge field one obtains the following hamiltonian [17]:

$$H = \frac{\rho_0}{2m} \sum_q \left(\frac{2\pi}{\nu q} \right)^2 \Pi_q \Pi_{-q} + q^2 \Theta_q \Theta_{-q} \quad (19)$$

where $\Pi_q \propto \delta \rho_q$ (the density fluctuations) is the canonical conjugate of Θ_q (compare with (3)) and whose exact ground state is Laughlin wavefunction at filling $\nu = 1/(2n + 1)$. (v) For fermions, in both instances, reduction to the gaussian theory is done after a change in statistics (CS transformation in 2D, Jordan-Wigner transformation in one dimension): we then end up with Landau-Ginzburg theories for hard-core bosons (the composite bosons of the FQHE in two dimensions). (vi) Both hamiltonians exhibit a superfluid rigidity, with a quasi Off-Diagonal Long-Range Order (ODLRO) for the Laughlin bosons (as stressed in two dimensions in [28]): it is easily seen from the boson hamiltonians that $\langle \Psi_B(0) \Psi_B^\dagger(x) \rangle \sim 1/|x|^{1/2K}$ (with K replaced by ν in 2D). The LL is a critical theory with algebraic decay for all order parameters. Note however that for attractive interactions ($K > 1$) superfluidity of Laughlin bosons is always the dominant order parameter, while for repulsive interactions ($K < 1$), it is superfluidity for the dual variable (vortices) which then has the slower decay ($\langle e^{i\sqrt{\pi}\phi(0)} e^{i\sqrt{\pi}\phi(x)} \rangle \sim 1/|x|^{K/2}$). Therefore either one or the other of these two superfluidities is always the dominant order. (vii) Charged excitations obey fractional exclusion statistics [24]. (viii) When we consider multi-component systems, in both cases again, there will be a quantum-numbers separation.

We then define Jastrow-Luttinger Fractional Liquids to be superfluids with Jastrow-Laughlin ground states,

Laughlin quasi-particles carrying anomalous charges, Bijl-Feynman collective modes and displaying a quantum number separation. A striking point when we consider fermions is that the ODLRO characteristic of superfluidity is *hidden*: it appears after a singular gauge transformation converting the statistics of fermions into a bosonic one (Chern-Simons or Jordan-Wigner transmutation). This peculiar hidden order was first emphasized by Girvin and MacDonald in [28] for the FQHE and explains the strong analogy with the physics of ^4He (Jastrow ground states, vortices, phonons, rotons): as shown above that property is very remarkably shared by the LL which we re-interpret as a hard-core boson superfluid.

We now prove spin-charge separation for the FQHE; as early as 1983, Halperin introduced the following two-component wave functions [8]:

$$\phi_{m,m',n} = \prod_{i < j} (z_{ij})^{g_{\sigma_i, \sigma_j}} \prod_i \exp -\frac{|z_i|^2}{4} \quad (20)$$

with $\hat{g} = \begin{pmatrix} m & n \\ n & m' \end{pmatrix}$ in order to describe non-fully polarized states in the QHE at fillings $\nu = (m + m' - 2n)/(mm' - n^2)$. Such wavefunctions are Laughlin's state natural extensions to multicomponent systems and have been widely used to describe spin effects and multilayer systems. For $m = m' = n + 1$, we have singlet states [29]. As in the single component case these wavefunctions can be derived microscopically from bosonic Chern-Simons Landau-Ginzburg (CSLG) theories; for instance one may check that $\phi_B = |\phi_{m,m,n}|$ is the exact ground state of:

$$H = \frac{\rho_0}{2m} \sum_{q,\sigma} \left(\frac{2\pi}{q} \right)^2 (m+n)^2 \Pi_{q,\sigma} \Pi_{-q,\sigma} + 4mn \Pi_{q,\uparrow} \Pi_{-q,\downarrow} + q^2 \Theta_{q,\sigma} \Theta_{-q,\sigma} \quad (21)$$

at filling $\nu = 2/(m+n)$ for $m \neq n$ [30]. (Following eq.(12) a similar generalization to the case $m \neq m'$ can also be written.) From the previous analogous discussion of the LL, it should be clear however that $\phi_{m,m',n}$ and H for $m = m'$ are both spin-charge separated! More generally, again there will be a quantum numbers separation, with a separation for the hamiltonian, the ground state and all observables: the quantum numbers of the electron have vanished from the excitation spectrum.

A possible worry in the FQHE is the requirement of lowest Landau level projection: it is well known that CSLG theories do not adequately describe the neutral modes since they do not work with projected density operators. Still, using a full first quantized approach to multicomponent FQHE systems removes the problem [31,29]: the ground state is taken as $\phi_{m,m',n}$, charged excitations are $\prod_i (w_i - z_0) \phi_{m,m',n}$ or $\prod_i (y_i - z_0) \phi_{m,m',n}$ (with notations similar to the 1D case), and neutral excitations are found which for $m = m'$ are precisely $P\rho_c \phi_{m,m',n}$

and $P\rho_s \phi_{m,m',n}$, i.e. charge and spin modes again (P is the lowest Landau level projector). This shows that lowest Landau level projection is immaterial to the issue of spin-charge separation.

We note that Laughlin quasiparticles are the only topologically allowed charged excitations: in analogy to the LL we propose to define Q and J as the number of quasiparticles and the circulation of a vortex (in units of ϕ_0); integer valuedness of these topological charges is only realized for Laughlin quasiparticles. More precisely we write Laughlin quasiparticles as 2D vertex operators: $V_{Q,J}^{2D}(z_0) = \prod_i [(z_i - z_0) / |z_i - z_0|]^J |z_i - z_0|^Q$ (a difference with the 1D case is that $Q = J$ to implement analyticity). Then all the considerations made above for the charged excitations of a LL can be adapted to the FQHE with the formal replacement: $z = \exp i \frac{2\pi}{L} x \rightarrow z = x + iy$. For instance $(e_\uparrow, e_\downarrow) = \left(\frac{m'}{mm' - n^2}, \frac{-n}{mm' - n^2} \right)$ for $\prod_i (w_i - z_0) \phi_{m,m',n}$ which is associated with the 2D vertex operator $V_{1100}^{2D} = \prod_i (w_i - z_0)$; more generally, the charges carried are recovered as $e_\sigma = g_{\sigma\tau}^{-1} Q_\tau$ for $V_{Q_\sigma, J_\sigma}^{2D}$. Again it is clear that the quasiparticle wavefunctions can be written as products of independent parts as in eq.(18) (charge and spin parts if $m = m'$) and with a pseudo-confinement. In other words, for $m = m'$ (but $m \neq n$ which is a special case due to spin degeneracy -see below) we find that holon and spinon excitations exist although the system is *two* dimensional; a difference with the LL however is that both excitations are gapped. Explicitly, the holon and spinon wavefunctions are respectively: $h(z_0) \phi_{m,m,n} = \exp \frac{1}{2} \int \rho_c(x) \ln(z - z_0) dx \phi_{m,m,n}$ and $s(z_0) \phi_{m,m,n} = \exp \frac{1}{2} \int \rho_s(x) \ln(z - z_0) dx \phi_{m,m,n}$ (these expressions show explicitly the independence of holons and spinons if the system is indeed described by a boson hamiltonian with a separation in charge and spin densities -as in (21)). The role of the LL parameter K is played by the filling factor ν ; for instance in the spin-charge separated case $K_\rho \iff (\nu_\uparrow + \nu_\downarrow)/2$ and $K_\sigma \iff (\nu_\uparrow - \nu_\downarrow)/2$. Charge and spin of the holon and spinon are as in 1D after such formal replacements in the previous formulas. As in the 1D case holons and spinons are free although they appear together in the Laughlin quasi-hole wavefunction $\prod_i (w_i - z_0) \phi_{m,m,n} = h(z_0) s(z_0) \phi_{m,m,n}$ due to the constraint on the total topological charges Q_σ and J_σ which must be integer valued.

We observe also that the charge and spin parts of the wavefunctions of the ground state and of the charged excitations (e.g. $h(z_0)$) are not analytical functions, in violation of lowest Landau level projection: this is no cause for concern because of pseudo-confinement. Indeed the only requirement is that the full wavefunction be analytical; by virtue of pseudo-confinement the spin and charge parts (spinons and holons) are never observed separately although they are still independent because of spin-charge separation. In short, the description of the multicomponent FQHE -either in first quantization or in

a CSLG approach- supports quantum numbers separation in 2D in full parallel to the LL.

We expect that analysis to break down if there is a degeneracy of the ground state because of the possibility of novel excitations (textures). And indeed for $\phi_{m,m,m}$ (i.e. $m = m' = n$) which describes a QH ferromagnet [29] we obtain a singular (non invertible) \hat{g} matrix. But the filling fractions ν_σ , the charges of the Laughlin quasiparticles e_σ and therefore their spin S_z are determined by \hat{g}^{-1} and therefore are ill-defined: this just reflects the spin degeneracy. That degeneracy means that the above description for the excitations of the system is insufficient since we can consider textures interpolating between two ground states: this is precisely the skyrmion excitation introduced for QH ferromagnets. Since this lies out of the above theoretical framework there is no reason to demand spin-charge separation.

Turning to experiments, we look for fillings suitable to test quantum-numbers separation; it is known that for many fillings the quantum Hall state is not fully polarized [33]: this is explained by the small effective masses and Zeeman couplings observed in real samples, which can increase the ratio of the cyclotron to the Zeeman gap up to a factor of 50 (as in *GaAs*). Most promising for the observation of spin-charge separation are fillings such as $\nu = 8/5$ or $2/3$ for which experimental evidence points to non-polarized ground states (although they are not inconsistent with partially polarized ones) as seen in tilted-field experiments [33] (where transitions to polarized states are observed as the in-plane field increases). Further support for a non-polarized ground state at $\nu = 8/5$ comes from numerical results at the particle-hole conjugate filling $\nu = 2/5$ (identical ground states are expected after particle-hole conjugation): the exact ground state was shown to be unpolarized for small values of the Zeeman coupling g , while Halperin's unpolarized $\phi_{3,3,2}$ was also shown to have a very good variational energy (better than the fully polarized hierarchical state), although unfortunately its overlap with the true ground state was not examined [32].

How could spin-charge separation be experimentally tested? Given that separation in the bulk automatically entails separation at the edges, experiments at the edge would confirm quantum-numbers separation for edges and provide strong support in favor of its existence in two dimensions (reflecting our claim that ground state wavefunctions have identical functional forms both at the edges and in the bulk). Various tests were proposed for the specific case of the LL: probing the spectral density which has a two-peak structure due to spin-charge separation [36], spin injection which probes spin transport [37]. In a different context, for $\nu = 2/3$ hierarchical spin polarized edges, where some theories predict a neutral collective mode besides a charged one, Kane and Fisher suggested time domain experiments, in which an electron is injected through a tunnel junction at the

edge of a FQHE disk [38]. This proposal evidently applies for the detection of charge and (real) spin modes since the latter is neutral. Upon injection of the electron, the collective charge and spin modes are excited but since they propagate at different speeds a detector (another tunnel junction) would see two pulses. If we now replace the junction by a capacitor, one will detect only the charge mode thereby proving the existence of a neutral mode and of spin-charge separation, which is relevant for such fillings as $\nu = 8/5$.

Edge states in the FQHE.

We now turn to a discussion of edge states in the FQHE, where the strong unity between the LL and the FQH liquid as Jastrow-Luttinger Fractional liquids is further evidenced. At the edge of QH samples, gapless states must exist since the Fermi level crosses the confining potential [34]. On general grounds (CS theories and hydrodynamical theories) Wen argued convincingly that these edge states are described by a chiral LL [3]. The relation to Laughlin microscopic approach remained obscure however. Let us try to see how this may come about: we will make use of two operations, one dimensional restriction of the bulk wavefunction and analyticity for lowest Landau level states. (The first operation can be given a precise meaning as is well known under the namesake of one dimensional reduction in the lowest Landau level, or from a direct reduction of the microscopic hamiltonian. See [35] and references therein. For our purposes, the following physical argument will be enough: since bulk excitations lie above a gap, while edge states are gapless, the latter decouple dynamically from the bulk for sufficiently low-energy processes: bulk and edge can be thought of as living in two decoupled Hilbert spaces, which gives meaning to a restriction of bulk wavefunctions to the edges.)

From the above discussion of the LL, the relation between Laughlin microscopic variational approach and the edge theories should be intuitively clear: heuristically, in the bulk we have a Laughlin wavefunction and therefore at the edge we also have a Jastrow-Laughlin wavefunction, which is precisely the variational wavefunction associated with the LL; consequently we have a LL at the edges, chirality stemming from the magnetic field. However it must be noted that the bulk wavefunction is analytic while the fermionic Luttinger wavefunction we gave above was not. This is easily remedied by generalizing our previous treatment of the LL as follows: an implicit assumption for the fermion fields $\Psi = \exp(i\sqrt{\pi}\Theta)$ ($\exp(ik_F r + i\sqrt{\pi}\Phi) + \exp(-ik_F r - i\sqrt{\pi}\Phi)$) was made, namely that they had conformal spin $1/2$ (conformal spin for operators $V_{\alpha\beta}$ is $S = \pi\alpha\beta/2$; after a Wick rotation, it describes the usual spin in Euclidean space-time); we may consider however higher spins such as $S = (2n + 1)/2$. This can be done by "flux" attachment (iteration of Jordan-Wigner transformations): $\Psi' = \exp(i\sqrt{\pi}2n\Phi)\Psi$, or by a modifica-

tion of the relations between particle currents and the fields: $\delta\rho = \nabla\Phi/\sqrt{\pi(2n+1)}$ and $j = -\nabla\Theta/\sqrt{\pi(2n+1)}$ (in technical CFT jargon, we have modified the $U(1)$ Kac-Moody relations which define the LL) with the new definitions $\Psi = \exp(i\sqrt{\pi(2n+1)}\Theta)$ ($\exp(ik_F r + i\sqrt{\pi(2n+1)}\Phi) + h.c.$). The first operation simply leads to the following fermionic ground state: $\psi_0^F(\{r_1, \dots, r_N\}) = \prod_{i < j} (z_{ij}/|z_{ij}|)^{2n+1} |z_{ij}|^{1/K} \exp(ik_F \sum r_i + c.c.)$ which under a 2π rotation of z coordinates yields indeed a phase $4\pi S$ with $S = (2n+1)/2$; we will follow the second prescription which is relevant for edge states: by retracing the steps taken to obtain equ.(6) the second operation is easily shown to lead to $\psi_0^F(\{r_1, \dots, r_N\}) = \prod_{i < j} (z_{ij}/|z_{ij}|)^{2n+1} |z_{ij}|^{2n+1/K} \exp(ik_F \sum r_i + c.c.)$.

If we restrict now the 2D $\nu = 1/(2n+1)$ Laughlin bulk ground state to 1D, this corresponds to the ground state of a LL with modified Kac-Moody relations if we choose $K = 1$ and if we drop the anti-analytical part of the wavefunction, i.e. if the LL is chiral. Indeed implementing the analyticity constraint on the excitations of the full non-chiral LL means that we have to restrict to $q > 0$ in (6) and to $\alpha = \beta$ for the $V_{\alpha,\beta}$ vertex operators. This then means that these excitations are chiral since $q > 0$ while $\alpha = \beta$ implies that one of the chiral charges is always zero. We observe therefore that lowest-Landau level analyticity requirement and one-dimensional reduction alone imposed on wavefunctions allow one to recover a chiral LL. We also note that the operator whose action is $\prod_i (z_i - z)$ is $V_{\alpha,\beta}$ with $\alpha = \beta = 1/\sqrt{(2n+1)}$. It has therefore anyonic statistics $\pi/(2n+1)$ as expected (that operator is allowed if we are on an annulus). (The operation of flux attachment we first described above as an alternative to modifying Kac-Moody relations can be shown to lead to a gapless fractionally charged excitation, which is not allowed for a single FQH edge on a disk geometry since Laughlin quasiparticles are gapped excitations; this is the reason why we have considered the second alternative.)

Spin liquids.

We now want to point out an interesting result for quantum magnetism: if we compute the spin correlators for Halperin's $\phi_{m+1,m+1,m}$ which is a $SU(2)$ spin singlet, we find from the gaussian hamiltonian - eq.(21) - that the longitudinal (and due to rotational invariance, transverse as well) spin-spin correlators behave as $\langle S_q^z S_{-q}^z \rangle \sim q^2$ showing that spin-spin correlations are suppressed. We have a local singlet state for which spins are completely screened (this can be seen by exploiting spin-charge separation: the spin part was studied in [39] and represents in the plasma analogy a two component neutral Coulomb gas in the disordered phase of the Kosterlitz-Thouless transition). We have therefore a (spin-charge separated) spin liquid! This supports Kalmeyer-Laughlin proposal of using Laughlin-like wavefunctions to describe spin-liquids [40]. We note that we have thus two classes of

spin liquids since in one dimension gapless spin liquids are also described by Jastrow-Laughlin wavefunctions. We stress the non-triviality of these statements: apart from exact Bethe Ansatz states, very few fundamental magnetic states are known for quantum antiferromagnetism. Indeed the Néel state (and its Ising variant) stand out almost alone. We now have in low dimensions to add to that list the Laughlin state. We note also that these spin-liquids are Mott-Hubbard insulators: in the bulk, the $\phi_{m+1,m+1,m}$ which is a state induced by the repulsive Coulomb interactions is clearly insulating (there is a gap to charged excitations and a zero conductivity), with the additional peculiarity that it is a spin charge separated Mott-Hubbard insulator.

We summarize the novel results obtained in this paper: we have shown that the Luttinger liquid can be understood in terms of Jastrow-Laughlin states, Laughlin quasiparticles, and Bijl-Feynman phonons. As in the FQHE anomalous charges are sustained by the LL. We have introduced the concept of pseudo-confinement for condensed-matter systems and generalized spin-charge separation to two dimensions for the FQHE (for situations without spin-degeneracy). We have characterized the LL and the FQH fluid as Jastrow-Luttinger Fractional Liquids, i.e. superfluids with a hidden off-diagonal long-range-order for Laughlin bosons.

In low dimensions interactions and quantum fluctuations conspire to stabilize novel quantum liquids : in that respect Jastrow-Luttinger liquids which comprise the Luttinger liquid and the FQH fluid are paradigmatic of strongly correlated systems. Such systems describe liquids which develop Jastrow-Laughlin correlations to minimize interactions leading to a novel kind of quantum coherence precisely described in a mean-field like manner by Jastrow-Luttinger states. This special coherence means that the spectrum of excitations is entirely collective: usual quasiparticles have vanished (orthogonality catastrophe) through a quantum numbers separation and through fractional charged excitations describing density bumps and holes above the ground state sea. Fractionalization is complete. Such liquids are astonishingly diverse: there are strange non-Fermi liquids metals yet with a Fermi surface, Mott-Hubbard insulators or spin liquids. We end up with a remark on high-temperature superconductivity: while the ideas of a spin-liquid and of spin-charge separation proposed by Anderson [41] are quite controversial issues, we believe that this work contributes to validate these concepts theoretically (and in the near future hopefully experimentally) in two dimensions, albeit in the different context of Jastrow-Luttinger Fractional Liquids. Whether the paradigm set by Jastrow-Luttinger fractional liquids will eventually prove useful for the understanding of high-temperature superconductivity remains an open question. The authors thank B.Jancovici, D.Bazzali for interesting discussions, as well

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Appendix: statistical transmutations.

We show that the Jordan-Wigner transformation is just the 1D restriction of the 2D Chern-Simons transformation. The CS operator reads:

$$U(z) = \exp i \int dz' \hat{\rho}(z') \arg(z - z')$$

where $\arg(z)$ is the argument of the complex variable z . Applied on a fermionic operator, it transforms it into a hard-core boson. If z and z' are constrained to lie on the real line, then $\arg(z - z') = \pi\theta(x - x')$ where θ is the step function. $U(z)$ becomes:

$$\begin{aligned} U(x) &= \exp i \int dx' \hat{\rho}(x') \pi\theta(x - x') \\ &= \exp i\pi \int_{-\infty}^x dx' \hat{\rho}(x') \end{aligned}$$

which is just the continuum version of the usual Jordan-Wigner transformation:

$$U_n = \exp i\pi \sum_{j < n} c_j^\dagger c_j$$

In the bosonization formalism, $U(x) = \exp i(k_F x + \sqrt{\pi}\phi(x))$ since $\hat{\rho}(x) = k_F/\pi + \nabla\phi/\sqrt{\pi}$. In first quantization the action of both $U(z)$ and its 1D restriction $U(x)$ are simply to multiply the wavefunction by the phase factor $\Pi_i(z_i - z)/|z_i - z|$. We note also that spins 1/2 are hard-core bosons; therefore they can be fermionized in 1D and 2D by using Jordan-Wigner or Chern-Simons transformations.

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